Ramberg-Osgood Elasticity Property for OOF

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This software package provides a particular form of nonlinear elasticity property for the OOF software. It is written as an OOF internal extension, so it is not loaded by default when oof is run. See the manual for more on OOF extensions.

1 Constitutive Rule

Ramberg-Osgood elasticity is a form of nonlinear elasticity designed to emulate plastic yield. Unlike true plasticity, there is no stress-free strain in the Ramberg-Osgood scheme — deformation following this rule is fully recovered when the load is removed.

The strain as a function of the stress is given by

$$\varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} + \frac{3}{2}\alpha\left(\frac{q}{s_0}\right)^{n-1}(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}) \tag{1}$$

where ε_{ij} is the strain tensor, σ_{ij} is the stress tensor, E and ν are the Young's modulus and Poisson ratio of the material, s_0 is a reference stress value, which may be usefully thought of as the yield stress. α is a constitutive parameter with dimensions of inverse stress, and q is the Von Mises equivalent stress, given by

$$q = \sqrt{\frac{3}{2}(\sigma_{ij}\sigma_{ij} - \frac{1}{3}\sigma_{kk}^2)}$$

This can also be expressed slightly more awkwardly in terms of the tensor elastic constants, which are more directly available in OOF:

$$\varepsilon_{ij} = A\sigma_{ij} - B\sigma_{kk}\delta_{ij} + \frac{3}{2}\alpha\left(\frac{q}{s_0}\right)^{n-1}(\sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}) \tag{2}$$

with

$$A = \frac{1}{c_{11}-c_{12}} \qquad \text{and} \qquad B = \frac{c_{12}}{(c_{11}-c_{12})(c_{11}+2c_{12})}. \tag{3}$$

The first two terms of the constitutive rule, Eq. 2, are those of conventional linear elasticity. The third term provides for potentially rapid increases in strain

for modest increases in the shear part of the stress, which is characteristic of a yielding material.

2 The OOF Property

This OOF property uses the constitutive rule above, and provides for user input of the parameters α, n, and s_0, along with the isotropic linear elastic parameters in whatever form the user prefers.

The architecture of the OOF software is such that properties are asked to specify their fluxes in terms of the fields being solved for in the computation. For elasticity, this means that constitutive rules should give the stress as a function of the strain.

Obviously, the constitutive rule in Eq. 2 does not do this. In the present implementation, what the property does is, for each evaluation point, first determine the local value of the strain, and then numerically compute the local stress by solving Eq. 2 via a Newton-Raphson scheme. Once the local stress is known, the property can then compute the local compliance matrix, which is the derivative of Eq. 2 with respect to stress. The local stiffness matrix is then the inverse of this matrix. An additional stress offset is also computed, so that what is finally returned to the OOF software is the best local linear approximation to the constitutive rule, with the independent variable being the strain.

The OOF code then takes care of solving this problem using an appropriate nonlinear solver, iterating the system as a whole to convergence.

3 Building

First, make sure that OOF2 is built and installed.

The extension source code and a setup.py script are located in the ramberg-osgood directory of the OOFEXTENSIONS package. This code requires SWIG version 1.1 build 883 in order to work. Unlike the OOF distribution, it is not possible to skip running SWIG when building this extension. If a functional OOF2 installation is present on the system, and SWIG is available, it should be sufficient to cd to the directory of the setup script, and run:

```
> python setup.py install --prefix=<oof-installation-target>
```

The installation target should be the same one that was used when installing the main OOF program. If this target is not specified, it defaults to /usr/local.

Once installed, the property will not automatically be loaded into the OOF software. To accomplish this, either start OOF2 with the `--import` command line argument, like this:

```
oof2 --import ramberg_osgood
```

or after it's started, open an OOF console (or run OOF in text mode) and type:

```
>>> import ramberg_osgood
```

The Ramberg-Osgood elasticity property should then appear in the list of available properties under Mechanical and Elasticity in the property pane on the OOF materials page.

4 Disclaimer